

**FACULTY OF SCIENCE****DEPARTMENT OF PURE AND APPLIED MATHEMATICS**

MODULE	MAT01A1 CALCULUS OF ONE-VARIABLE FUNCTIONS
CAMPUS	APK
ASSESSMENT	EXAM

DATE 30/05/2017**TIME 12:30****ASSESSOR(S)****DR A CRAIG**
MS S RICHARDSON**INTERNAL MODERATOR****MR S MAFUNDA****DURATION 2 HOURS****MARKS 70****SURNAME AND INITIALS** _____**STUDENT NUMBER** _____**CONTACT NUMBER** _____**NUMBER OF PAGES: 1 + 12 PAGES**

INSTRUCTIONS:

- 1. ANSWER ALL THE QUESTIONS ON THE PAPER IN PEN.**
- 2. NO CALCULATORS ARE ALLOWED.**
- 3. SHOW ALL CALCULATIONS AND MOTIVATE ALL ANSWERS.**
- 4. IF YOU REQUIRE EXTRA SPACE, CONTINUE ON THE ADJACENT BLANK PAGE AND INDICATE THIS CLEARLY.**

Question 1 [8 marks]

For questions 1.1 – 1.8, choose **one** correct answer, and make a cross (X) in the correct block.

Question	a	b	c	d	e
1.1					
1.2					
1.3					
1.4					
1.5					
1.6					
1.7					
1.8					

1.1 Find the vertical asymptotes of the function $y = \frac{2x^2 + 1}{3x - 2x^2}$. (1)

- (a) $x = 0$ only
- (b) $x = \frac{2}{3}$ only
- (c) $x = 0, x = \frac{2}{3}$
- (d) $x = \frac{3}{2}, x = 0$
- (e) None of the above

1.2 Find the point(s) on the graph of $f(x) = x^3 - 2$ where the slope is 3. (1)

- (a) $(\sqrt[3]{2}; 0)$
- (b) $(1; 3)$ and $(-1; 3)$
- (c) $(1; -1)$ and $(-1; -3)$
- (d) $(1; 3)$
- (e) None of the above

1.3 Suppose $f(3) = 0$, $f'(3) = 6$, $g(3) = 1$, $g'(3) = \frac{1}{3}$, and $h(x) = \frac{f(x)}{g(x)}$. Evaluate $h'(3)$. (1)

- (a) 18
- (b) 6
- (c) -6
- (d) -2
- (e) None of the above.

1.4 Let $f(x) = \frac{x^2 - x - 12}{x - 4}$. How would you define $f(4)$ to make f continuous at $x = 4$? (1)

- (a) $f(4) = 0$
- (b) $f(4) = 3$
- (c) $f(4) = 4$
- (d) $f(4) = 7$
- (e) None of the above

1.5 If $2x - 1 \leq f(x) \leq x^2$ for $0 < x < 3$, find $\lim_{x \rightarrow 1} f(x)$. (1)

- (a) 1
- (b) -1
- (c) 0
- (d) 3
- (e) None of the above

1.6 Find $f(x)$ if $f'(x) = \sin x$ and $f(\pi) = 2$. (1)

- (a) $f(x) = \cos x + 1$
- (b) $f(x) = -\cos x + 3$
- (c) $f(x) = -\cos x + 1$
- (d) $f(x) = \cos x + 3$
- (e) None of the above

1.7 $\sum_{n=1}^{20} (-1)^n$ is equal to: (1)

- (a) 0
- (b) 20
- (c) -20
- (d) 1
- (e) None of the above

1.8 The solution of $2 \ln(-x) = 1$ is: (1)

- (a) There are no solutions
- (b) $e^{-\frac{1}{2}}$
- (c) $-e^{\frac{1}{2}}$
- (d) $\frac{1}{2}$
- (e) None of the above

Question 2 [3 marks]

Solve the inequality: $\frac{-3}{x^2 + 4x} < 1$. (3)

Question 3 [2 marks]

Prove the identity : $\tan x + \tan y = \frac{\sin(x + y)}{\cos x \cos y}$ (2)

Question 4 [5 marks]

(a) If $u(x) = \tan^2(x^2 + 2)$ find $f(x), g(x), h(x)$ and $j(x)$ such that $u(x) = (f \circ g \circ h \circ j)(x)$. (2)

(b) Given $f(x) = \ln(-3 + \ln(2x))$

(i) Find the domain of f . (1)

(ii) Find the inverse function f^{-1} . (2)

Question 5 [4 marks]

Calculate the following limits **without** using L'Hospital's rule:

(a) $\lim_{t \rightarrow 2} \frac{t^2 - 4}{t^3 - 8}$ (2)

$$(b) \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 - 9}}{2x - 6} \quad (2)$$

Question 6 [4 marks]

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function and $a \in \mathbb{R}$. Prove the following statement: (4)

If f is differentiable at a then f is continuous at a .

Question 7 [3 marks]

Use the definition of the derivative at a point to calculate $f'(6)$ if $f(x) = \sqrt{x-2}$. (3)

Question 8 [9 marks]

Find the following derivatives:

(a) $\frac{dy}{dx}$ if $y = \sqrt{4 + e^{2x}} + \sec x$ (2)

(b) $\frac{dy}{dx}$ if $\cos 3y = \tan 2x$ (2)

(c) $D_x \left[\frac{x^3}{3^x} \right]$ (2)

(d) $\frac{dy}{dx}$ if $y = -\coth \frac{1}{x} + \ln \sqrt{\tanh 2x}$ (3)

Question 9 [3 marks]

Show that the function $f(x) = x - \cos x$ has a root in the interval $(0, \pi)$. (3)

Question 10 [3 marks]

Find the limit: $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{2}{n} \left(1 + \frac{2i}{n}\right)^2$ (3)

Question 11 [2 marks]

Evaluate: $\lim_{x \rightarrow \pi^+} \frac{\sin x}{\sqrt{x - \pi}}$. (2)

Question 12 [4 marks]

Prove that $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$. (4)

Question 13 [6 marks]

Evaluate the following integrals:

(a) $\int_0^1 \left(\frac{1}{1+x^2} + \frac{\pi}{4} \right) dx$ (2)

(b) $\int \left(5x^2 + \frac{1}{x} - \sin x \right) dx$ (2)

(c) $\int x \cos(4x^2) dx$ (2)

Question 14 [3 marks]

Complete the following table with the correct truth values: (3)

p	q	r	$p \wedge q$	$r \rightarrow q$	$\neg r$	$\neg p \vee q$	$q \rightarrow p$	$(\neg p \vee q) \leftrightarrow \neg r$
T			F		F			

Question 15 [4 marks]

(a) Write the statement “All real numbers are equal to two or strictly less than zero” as a first-order formula. (1)

(b) Is the statement in (a) true? Justify your answer. (1)

(c) Write the negation of the statement from (a) in natural language. (2)

Question 16 [3 marks]

Use proof by cases to show that $3n - n^2$ is even for all $n \in \mathbb{Z}$.

Question 17 [4 marks]

Prove that $11^n - 6$ is divisible by 5 for all $n \in \mathbb{Z}^+$.